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Heat transfer over an unsteady stretching surface with variable heat flux in the presence of a heat source or sink

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ABSTRACT

Unsteady boundary layer flow of an incompressible fluid over a stretching surface in the presence of a heat source or sink is studied. The unsteadiness in the flow and temperature fields is caused by the time dependence of the stretching velocity and the surface heat flux. The nonlinear boundary layer equations are transformed to nonlinear ordinary differential equations containing the Prandtl number, heat source/sink parameter and unsteadiness parameter. These equations are solved numerically by applying a shooting technique using the Runge–Kutta method. Comparison of the numerical results is made with previously published results under special cases, and the results are found to be in good agreement. Effects of the Prandtl number, heat source/sink parameter and unsteadiness parameter on the flow and heat transfer are studied.

1. Introduction

The problem of heat transfer from boundary layer flow driven by a continuous moving surface is of significance in a number of industrial manufacturing processes. Since the pioneering work of Sakiadis [1,2], various aspects of the problem have been investigated by many authors.

The steady boundary layer flow due to stretching with linear velocity was investigated by Crane [3]. Vleggaar [4] and Gupta and Gupta [5] have analyzed the stretching problem with constant surface temperature, while Soundalgekar and Ramana [6] investigated the constant surface velocity. Grubka and Bobba [7] have analyzed the stretching problem for a surface moving with a linear velocity and with a variable surface temperature. Ali [8] has reported flow and heat characteristics on a stretched surface subject to power-law velocity and temperature distributions. The flow field of a stretching wall with a power-law velocity variation was discussed by Banks [9]. Ali [10] and Elbashbeshy [11] extended Banks's work to a porous stretched surface with different values of the injection parameter.

In all the previous investigations, the effects of an internal heat source or sink on the heat transfer were not studied. When there is an appreciable temperature difference between the surface and the ambient fluid, one needs to consider the temperature-dependent heat source or sink which may exert strong influence on the heat transfer characteristics. Foraboschi and Federito [12] assumed the volumetric rate of heat generation as

$$Q = \begin{bmatrix} Q_0(T - T_0) & \text{if } T > T_0 \\ 0 & \text{if } T < 0. \end{bmatrix}$$

Elbashbeshy and Bazid [13] studied the heat transfer over a stretching surface with internal heat generation. Ali [14] investigated the effect of lateral mass flux on the natural convection boundary layer induced by a heated vertical plate

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embedded in a saturated porous medium with internal heat generation. The unsteady heat transfer problem over a stretching surface, which is stretched with a velocity that depends on time, is considered by Anderson et al. [15], Elbashbeshy and Bazid [16] and Ishak et al. [17]. The present work studies heat transfer over an unsteady stretching surface with variable heat flux in the presence of a source/sink.

2. Formulation of the problem

Consider the unsteady two-dimensional laminar boundary layer flow of an incompressible fluid over a continuous moving stretching surface. It is assumed that the surface is stretched with velocity $U_w(x,t) = \frac{\alpha x}{1-\gamma t}$ along the x axis, keeping the origin fixed; the y axis is normal to the x axis. Also it is assumed that the surface is subjected to a variable heat flux $q_w(x,t) = \frac{bx}{1-\gamma t}$.

The governing basic boundary layer equations for momentum and energy take the following form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} + \frac{Q}{\rho c_p}(T - T_\infty) \tag{3}$$

subject to the boundary conditions

$$\begin{aligned} &y = 0 : u = U_w, \qquad v = 0 \qquad \frac{\partial T}{\partial y} = -\frac{q_w}{k} \\ &y \to \infty : u = 0, \qquad T = T_\infty, \end{aligned} \tag{4}$$

where x and y represent coordinate axes along the continuous surface in the direction of motion and normal to it, respectively. u and v are the velocity components along the x and y axes, respectively, t is the time, ν is the kinematics viscosity, T is the temperature inside the boundary layer, c_p is the specific heat at constant pressure, ρ is the density, $Q > 0$ represents a heat source and $Q < 0$ represents a heat sink, k is the thermal conductivity, T_∞ is the temperature far away from the stretching surface, and α, b and γ are constants, where $\alpha > 0$, $b \geq 0$, $\gamma \geq 0$ and $\gamma t < 1$. Both α and γ have dimension $(\text{time})^{-1}$.

The equation of continuity is satisfied if we choose a stream function $\psi(x,y)$ such that

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}.$$

The mathematical analysis of the problem is simplified by introducing the following dimensionless similarity variables:

$$\begin{aligned} \eta &= \sqrt{\frac{\alpha}{\nu(1-\gamma t)}}\,y \\ \psi(x,y) &= \sqrt{\frac{\alpha \nu x^2}{(1-\gamma t)}}\,f(\eta) \\ T &= T_\infty + \frac{q_w}{k}\left[\sqrt{\frac{\nu(1-\gamma t)}{\alpha}}\right]\theta(\eta). \end{aligned} \tag{5}$$

Substituting (5) into (2) and (3), we obtain the following set of ordinary differential equations:

$$f''' + ff'' - f'^2 - A\left(f' + \frac{1}{2}\eta f''\right) = 0 \tag{6}$$

$$\theta'' + Pr\left[f\theta' - f'\theta - \frac{A}{2}(\theta + \eta\theta') + \delta\theta\right] = 0. \tag{7}$$

The boundary conditions (4) now become

$$\begin{aligned} &\eta = 0 : \qquad f = 0, \quad f' = 1, \quad \theta' = -1 \\ &\eta \to \infty : \qquad f' = 0, \quad \theta = 0, \end{aligned} \tag{8}$$

where the primes denote differentiation with respect to η, $A = \frac{\gamma}{\alpha}$ is a parameter that measures the unsteadiness, $Pr = \frac{\mu c_p}{k}$ is the Prandtl number (μ is the viscosity), $\delta = \frac{Qk}{\mu c_p}\frac{\text{Re}_x}{\text{Re}_k^2}$ is the dimensionless heat source/sink parameter, $\text{Re}_x = \frac{U_w x}{\nu}$ is the local Reynolds number, and $\text{Re}_k = \frac{U_w\sqrt{k}}{\nu}$.

Table 1
Comparison of local Nusselt number at $A = 0$, $\delta = 0$ at different values of Pr with previously published data.

Pr	Ishak et al. [17]	Elbashbeshy [11]	Exact solution [18]	Present results
0.72	0.8086	0.8161	0.8086	0.808
1.0	1.0	1.0	1.0	1.0
10	3.7202	3.7202	3.7206	3.7207

Table 2
Results for the skin friction coefficient and the local Nusselt number for $Pr = 1$ at different values of the source/sink parameter and the unsteadiness parameter.

$Pr = 1$								
δ	-2		-0.1		0		0.1	
A	$f''(0)$	$1/\theta(0)$	$f''(0)$	$1/\theta(0)$	$f''(0)$	$1/\theta(0)$	$f''(0)$	$1/\theta(0)$
0	1	1.7844	1	1.0599	1	1	1	0.9322
0.8	1.3218	1.854	1.3218	1.1881	1.3218	1.1386	1.3218	1.0855
1.2	1.4535	1.8904	1.4535	1.2526	1.4535	1.207	1.4535	1.1589
2	1.6828	1.9635	1.6828	1.3742	1.6828	1.3345	1.6828	1.2933

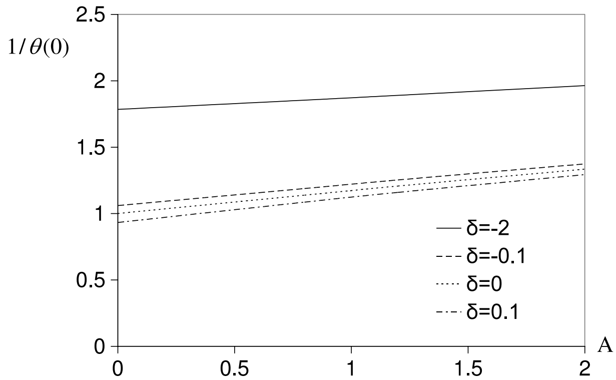

Fig. 1. Variation of the local Nusselt number for various values of δ at $Pr = 1$.

The physical quantities of interest in this problem are the skin friction coefficient C_f and the local Nusselt number Nu_x, which are defined as

$$C_f = \frac{\mu\left(\frac{\partial u}{\partial y}\right)_{y=0}}{[\rho U_w^2/2]}, \qquad Nu_x = -\frac{x\left(\frac{\partial T}{\partial y}\right)_{y=0}}{T_w - T_\infty}$$

$$C_f\sqrt{\mathrm{Re}_x} = f''(0), \qquad Nu_x/\sqrt{\mathrm{Re}_x} = -\frac{1}{\theta(0)},$$

where $\mathrm{Re}_x = \frac{xU_w}{\nu}$ is the local Reynolds number based on the surface velocity.

3. Results and discussion

It is worth mentioning that for $A = 0$ and $\delta = 0$ Eqs. (6) and (7) reduce to those of Elbashbeshy [11] and they reduce to those of Ishak et al. [17] when $\delta = 0$. In order to check the accuracy of the numerical solutions, a comparison of heat transfer characteristics at the surface for $A = 0$ and $\delta = 0$ at different values of Prandtl number are made with those of Elbashbeshy [11], Ishak et al. [17], and [18]. From Table 1, we note that there is a close agreement with these different approaches, and this verifies the accuracy of the method used.

From Table 2 and Fig. 1, we note that the unsteadiness parameter A increases the skin friction coefficient and the local Nusselt number, whereas the local Nusselt number decreases with increase the source/sink parameter. The effect of the unsteadiness parameter A on the velocity profile as a function of η for $Pr = 1$ is shown in Figs. 2a and 2b; we note that the velocity profile decreases with increasing value of A, which implies an increase in the skin friction coefficient. Fig. 3 gives the variation of temperature distribution within the boundary layer for various values of the heat source/sink parameter.

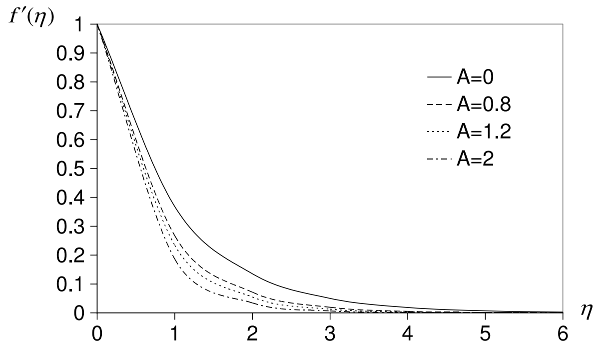

Fig. 2a. Velocity profiles for various values of A when $Pr = 1$ and $\delta = -2$.

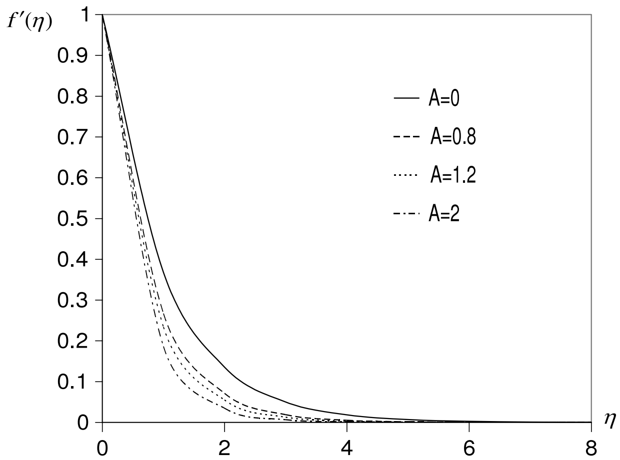

Fig. 2b. Velocity profiles for various values of A at $Pr = 1$, $\delta = 0.1$.

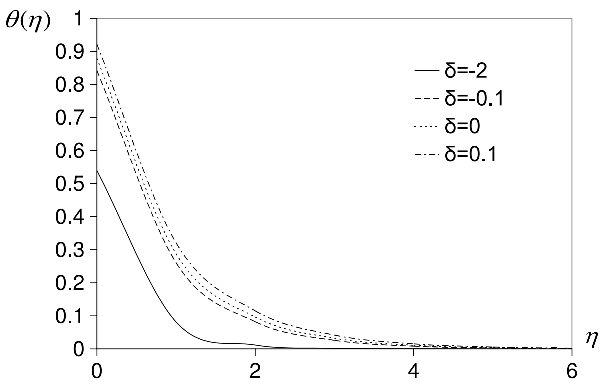

Fig. 3. Temperature profiles for various values of δ when $Pr = 1$ and $A = 0.8$.

The surface temperature $\theta(0)$ increases with increase in the heat source/sink parameter λ, which in turn decreases the local Nusselt number $\frac{1}{\theta(0)}$. Fig. 4 shows the temperature profiles at $Pr = 1$ for various values of A. From this figure it is seen that the temperature profile decreases with increasing unsteadiness parameter A. This shows that the rate of cooling is much faster for higher values of A while it may take a longer time for cooling during steady flow. Fig. 5 shows the temperature profiles for various values of the Prandtl number. The temperature decreases within the boundary layer for all values of the Prandtl number. This is consistent with the well-known fact that the thermal boundary layer thickness decreases with increasing Prandtl number.

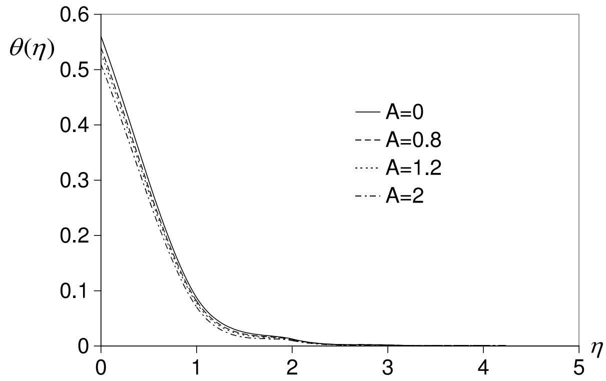

Fig. 4a. Temperature profiles for various values of A when $Pr = 1,\ \delta = -2$.

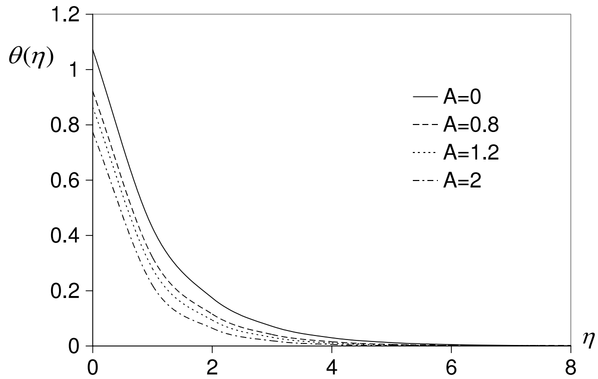

Fig. 4b. Temperature profiles for various values of A at $Pr = 1$ and $\delta = 0.1$.

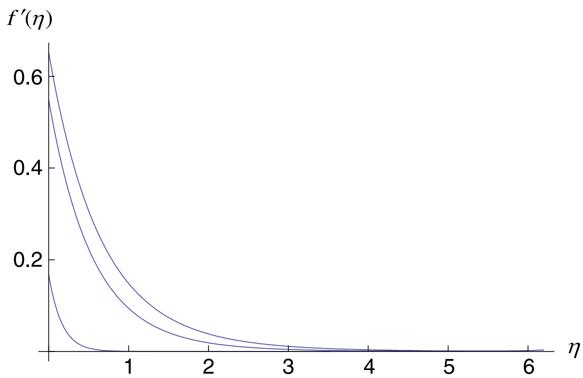

Fig. 5a. Temperature profiles for various values of Pr at $A = 0.8$ and $\delta = -2$.

4. Conclusion

A numerical method has been obtained to study the flow and heat transfer in the laminar flow of an incompressible fluid over an unsteady stretching surface. The effects of the unsteadiness parameter A, heat source/sink parameter λ and Prandtl number on the heat transfer characteristics were studied. The numerical results indicated the following.

1. The thickness of the momentum boundary layer decreases with increasing unsteadiness parameter A.
2. The unsteadiness parameter A increases the skin friction coefficient and the local Nusselt number.
3. Increasing the temperature decreases with an increase in the value of the unsteadiness parameter A.
4. Increasing the Prandtl number leads to a decrease in the surface temperature and thus an increase in the local Nusselt number.
5. Increasing the heat source/sink parameter λ leads to an increase in the surface temperature and thus a decrease in the local Nusselt number.

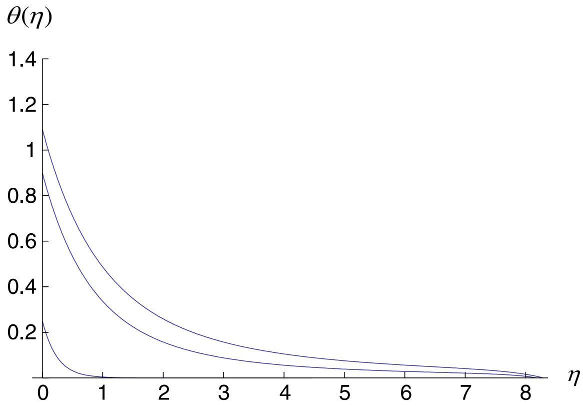

Fig. 5b. Temperature profiles for various values of Pr at $A = 0.8$ and $\delta = -0.1$.

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